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Adaptive Control of Quadrotor under Actuator Loss and Unknown State-dependent Dynamics

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***Abstract*— This paper examines the enhancement of quadrotor efficiency through adaptive Fault-Tolerant Control (FTC) in quadrotors amidst operational uncertainties and component inefficiency. State-of-the-art adaptive FTC strategies often assume the uncertainties to be bounded by a constant a priori, leading to ignorance of state-dependent uncertainties. Unattended state-dependent uncertainties that persist can lead to instability, especially under actuator faults. The proposed adaptive FTC offers actuator fault mitigation while tackling unknown (state-dependent) uncertainties via suitably designed adaptive laws. In addition, real-time fault detection and control allocation are used simultaneously to avoid conservative control application. The closed-loop system stability is studied analytically and the effectiveness of the proposed solution is verified on a realistic simulator in comparison to the state of the art.**

I. Introduction

Quadrotors have seen significant advancements in both applications and operational capabilities, from surveillance and delivery services to disaster response missions in recent times [1]–[3]. The efficacy of these systems critically hinges on the fault tolerance capabilities of their control mechanisms, especially in the face of actuator faults, for ensuring uninterrupted and safe operations [4], [5].

Fault-Tolerant Control (FTC) strategies can be broadly classified into passive and active approaches (cf. [6]–[8] and references therein). Passive FTC designs operate under predetermined fault conditions, avoiding real-time fault detection and isolation (FDI). Conversely, active FTC methodologies combine FDI with control strategies [6]. Hence, active FTC approaches have found more interests among the researchers owing to their real-time adaptability and comprehensive fault mitigation capabilities compared to passive FTC.

A. Related Works and Motivation

A foundational strategy in active FTC has been the modification of actuator control allocation matrix to accommodate faults [7], [8]. However, such methods overlook uncertainties and model imperfections. To bridge this gap, researchers have pivoted towards various robust (cf. [4], [9]–[13]) and adaptive control mechanisms (cf. [5], [6], [14]–[16]). Nevertheless, Adaptive controllers are advantageous over the robust controllers in tackling uncertainties with unknown bounds.

However, the adaptive controllers [5], [6], [14]–[16] can only tackle a priori bounded uncertainties. This leaves out the scope of state (acceleration or velocity)-dependent uncertainties from the inertial parameters (e.g., uncertainty in mass or in inertia due to change in center of mass). Ignoring this class of uncertainty not only impacts control performance adversely, but it can lead to unstable behavior (cf. [17]).

B. Contributions

In view of the above discussions, we propose an active adaptive FTC framework for quadrotors, under single/ multiple actuator faults having the following highlights:

- The proposed method can tackle unknown state-dependent model uncertainties without their a priori knowledge (unlike [5], [6], [14], [15]).
- The adaptive framework can be tuned to tolerate actuator loss-of-efficiency up to a user-defined limit.
- FDI and control allocation processes are utilized simultaneously with the adaptive control framework to avoid any conservative control application.

The closed-loop stability is analysed via the Lyapunov method. The effectiveness of the proposed method is extensively verified on a realistic simulation platform compared to the state of the art. Note that *'actuator loss'* in this work signifies actuator loss-of-efficiency, and not actuator 'failure' (i.e., when motors stop working). Some works (cf. [14], [18]) use redundant motors to study actuator failure case. However, discarding such structural redundancy, tackling complete loss of motors in quadrotors is very challenging due to increased underactuation. We plan to take such challenge in future work.

The rest of the paper is organised as follows: Sect. II describes the system dynamics and the control problem; Sect. III discusses the proposed adaptive controller design and analysis; Sect. IV presents the simulation results and Sect. V provides concluding remarks.

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The following notations are used in this paper: $||(\cdot)||$ and $\lambda_{\min}(\cdot)$ denote 2-norm and minimum eigenvalue of $(\cdot)$, respectively; I denotes identity matrix with appropriate dimension and $\text{diag}\{\cdot, \cdots, \cdot\}$ denotes a diagonal matrix with diagonal elements $\{\cdot, \cdots, \cdot\}$.

II. System Dynamics and Problem Formulation

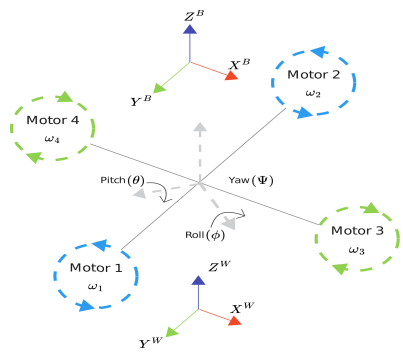

Fig. 1. Schematic of a quadrotor with coordinate systems

A. System Dynamics

The Euler-Lagrange (EL) dynamics of a quadrotor (cf. Fig. 1) under actuator Loss-of-Efficiency (LoE) is given by [15], [19],

$$m\ddot{p}(t) + G + d_p(t) = \gamma\tau_p(t), \tag{1a}$$
$$J(q(t))\ddot{q}(t) + C_q(q(t), \dot{q}(t))\dot{q}(t) + d_q(t) = \gamma\tau_q(t), \tag{1b}$$
$$\gamma\tau_p(t) = R_B^W(q(t))U(t), \tag{1c}$$

where (1a) and (1b) are the position and attitude dynamics of the quadrotor (cf. Table I for symbol definitions); $\tau_q \triangleq \begin{bmatrix} u_2 & u_3 & u_4 \end{bmatrix}^T \in \mathbb{R}^3$ denotes the control inputs for roll, pitch and yaw; $\tau_p \in \mathbb{R}^3$ is the generalized position control input in Earth-fixed frame, with $U \triangleq \begin{bmatrix} 0 & 0 & u_1 \end{bmatrix}^T \in \mathbb{R}^3$ being the force vector in body-fixed frame and R_B^W is the rotation matrix from the body-fixed coordinate frame to the Earth-fixed frame, given by

$$R_B^W = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & s_\phi c_\theta & c_\theta c_\phi \end{bmatrix}, \tag{2}$$

where $c_{(\cdot)}, s_{(\cdot)}$ and denote $\cos(\cdot), \sin(\cdot)$ respectively. Following the EL mechanics, the quadrotor dynamics (1) satisfies the following properties [19]:

Property 1. The inertia matrix $J(q)$ is uniformly positive definite $\forall q$ and there exist $\underline{j}, \overline{j} \in R^+$ such that $0 \leq \underline{j}I \leq J(q) \leq \overline{j}I$.

Property 2. There exist scalars $\overline{c}_q, \overline{d}_p, \overline{d}_q \in \mathbb{R}^+$ such that $||C_q(q, \dot{q})|| \leq \overline{c}_q||\dot{q}||$, $||d_q(t)|| \leq \overline{d}_q$, $||d_p(t)|| \leq \overline{d}_p$, $||G|| \leq \overline{g}_p$.

TABLE I

Nomenclature

$[X^B\ Y^B\ Z^B]$	Quadrotor body-fixed coordinate frame
$[X^W\ Y^W\ Z^W]$	Earth-fixed coordinate frame
$p = [x\ y\ z]$	Quadrotor position in $[X^W\ Y^W\ Z^W]$
$q = [\phi\ \theta\ \psi]$	Quadrotor roll, pitch and yaw angles
$m \in \mathbb{R}^+$	Mass of drone
$J, C_q \in \mathbb{R}^{3\times3}$	Inertia and Coriolis matrix
$G = [0\ 0\ mg]$	Gravity vector
$d_p, d_q \in \mathbb{R}^3$	Bounded external disturbance
$0 < \gamma \leq 1$	Loss-of-Efficiency (LoE) Parameter
ω_i	speed of i^{th} motor $(i = 1, \cdots, 4)$

Property 3. The matrix $(\dot{J} - 2C_q)$ is skew symmetric, i.e., any non-zero vector r yields $r^T(\dot{J} - 2C_q)r = 0$.

Remark 1 (LoE parameter)**.** *Since the quadrotor motion is governed by the combination of multiple motors, γ can be determined from the maximum allowable LoE as $\gamma = 1 - LoE$: for example, under 70% LoE we have $\gamma = 1 - 0.7 = 0.3$. We address the LoE at individual motor level during control allocation process subsequent to the control design (cf. Sect. III.C). The value $\gamma = 1$ denotes healthy actuators. Whereas, $\gamma = 0$ denotes uncontrollable state under one or multiple motor 'failure', and we avoid such condition in this work.*

Remark 2 (Desired Trajectories)**.** *The desired position $p_d = \begin{bmatrix} x_d & y_d & z_d \end{bmatrix}^T$ and desired yaw ψ_d are designed to be sufficiently smooth and bounded. The desired roll (ϕ_d) and pitch (θ_d) trajectories are derived using τ_p and ψ_d following the standard process outlined in [20]).*

Remark 3 (Uncertainty and Control Challenge)**.** *The terms m, J, C_q, d_p, d_q and their bounds $\overline{m}, \overline{c}_q, \overline{d}_p, \overline{d}_q$ are unknown for control design.*

Remark 4 (State-dependent Uncertainty)**.** *It can be noted from (1) that any uncertainty in m and J will lead to state (i.e. $\ddot{p}, \ddot{q}$)-dependent uncertain dynamics. Consideration of uncertainties to be bounded a priori by a constant ignores such cases (cf. [5], [6], [14], [15]). This work departs from the state-of-the-art methods in this direction and tackles such uncertainties.*

Assumption 1. *There exist control parameters $\gamma_m, E \in \mathbb{R}^+$ such that*

$$|\frac{\gamma}{\gamma_m} - 1| = E < 1 \tag{3a}$$
$$\implies \gamma_m > 0.5\gamma. \tag{3b}$$

The condition (3) provides an operational range for control design: for example, if one wants to make the controller feasible to tackle LoE of upto 60% (i.e., $\gamma = 0.4$), then γ_m should be designed to be $\gamma_m > 0.20$.

B. Loss-of-Efficiency Detection & Classification

Since developing control framework is the main focus of the work, we use a standard Support Vector Machine-based method as in [21] to assess motor LoE. A motor is deemed faulty if its speed consistently differs from the reference speed over a specified duration to avoid false alarms from sudden speed changes.

Let L_i, $i = \{1, \cdots, 4\}$ denote the the LoE of i^{th} motor. Furthermore, we define a diagonal matrix L as

$$L = \text{diag}\{(1 - L_1), (1 - L_2), (1 - L_3), (1 - L_4)\}. \quad (4)$$

The matrix L is used later during control allocation process in Sect. III.C.

Control Problem: Under Properties 1-3 and Assumption 1, to design an adaptive FTC for the quadrotor system (1a),(1b) under actuator Loss-of-Efficiency while tackling uncertainties described in Remark 3.

III. Proposed Adaptive Controller Design

This section discusses the proposed adaptive FTC framework, which consists of design of an outer loop adaptive controller for position dynamics (1a) (cf. Sect. III.A), an inner loop adaptive controller for attitude dynamics (1b) (cf. Sect. III.B) and control allocation process (cf. Sect. III.C). Subsequently, we shall remove variable dependency for brevity.

A. Outer Loop Controller

Let us define the position tracking error as $e_p \triangleq p - p_d$, and an error variable s_p as

$$s_p = \dot{e}_p + \Phi_p e_p, \quad (5)$$

where Φ_p is a positive definite matrix. Multiplying the time derivative of (5) by m and incorporating (1a) yields

$$m\dot{s}_p = m(\ddot{p} - \ddot{p}_d + \Phi_p \dot{e}_p) = \gamma \tau_p + \varphi_p, \quad (6)$$

where $\varphi_p \triangleq -(G + d_p + m\ddot{p}_d - m\Phi_p \dot{e}_p)$ is defined as the (state-dependent) uncertainty in position dynamics. Defining $\xi_p \triangleq \begin{bmatrix} e_p^T & \dot{e}_p^T \end{bmatrix}^T$ and using the inequality $||\xi_p|| \geq ||\dot{e}_p||$ and Property 2, the upper bound on φ_p can be derived as:

$$\begin{aligned} ||\varphi_p|| &\leq \overline{d}_p + \overline{g}_p + m(||\ddot{p}_d|| + ||\Phi_p|| ||\dot{e}_p||) \\ &\leq K_{p0}^* + K_{p1}^* ||\xi_p|| \end{aligned} \quad (7)$$

where $K_{p0}^* \triangleq \overline{g}_p + \overline{d}_p + m||\ddot{p}_d||$ and $K_{p1}^* \triangleq m||\Phi_p||$ are *unknown* finite scalars. The outer loop control law is proposed as:

$$\tau_p = \frac{1}{\gamma_m}\left(-\Lambda_p s_p - \rho_p \frac{s_p}{||s_p||}\right) \quad (8a)$$

$$\rho_p = \frac{1}{1 - E}(\hat{K}_{p0} + \hat{K}_{p1}||\xi_p||), \quad (8b)$$

where Λ_p is a user-defined positive definite gain matrix; $\hat{K}_{pi}$ is the estimation of K_{pi}^*, for $i = 0, 1$ computed through adaptive laws given by:

$$\dot{\hat{K}}_{pi} = ||s_p|| ||\xi_p||^i - \alpha_{pi} \hat{K}_{pi}, \ \hat{K}_{pi}(0) > 0 \quad (9)$$

with $\alpha_{pi} \in \mathbb{R}^+$ being user-specified design scalars. Since γ is not precisely known, U is computed from (1c) by replacing γ with γ_m.

B. Inner Loop Controller

The attitude error is derived in line with [20]

$$e_q = ((R_d)^T R_B^W - (R_B^W)^T R_d)^v \quad (10a)$$

$$\dot{e}_q = \dot{q} - R_d^T R_B^W \dot{q}_d, \quad (10b)$$

where $(.)^v$ denotes the *vee* map and R_d is the rotation matrix defined in (2) and evaluated at $(\phi_d, \theta_d, \psi_d)$.

Let us define an error variable s_q as

$$s_q = \dot{e}_q + \Phi_q e_q, \quad (11)$$

where Φ_q is a user-defined positive definite gain matrix. Multiplying the time derivative of (11) by J and using (1b), one can obtain

$$J\dot{s}_q = J(\ddot{q} - \ddot{q}_d + \Phi_q \dot{e}_q) = \gamma \tau_q - C_q s_q + \varphi_q, \quad (12)$$

where $\varphi_q \triangleq -(C_q \dot{q} + d_q + J\ddot{q}_d - J\Phi_q \dot{e}_q - C_q s_q)$ is the overall (state-dependent) uncertainties in attitude dynamics. Let us denote $\xi_q \triangleq \begin{bmatrix} e_q^T & \dot{e}_q^T \end{bmatrix}^T$. Then using the inequalities $||\xi_q|| \geq ||e_q||$, $||\xi_q|| \geq ||\dot{e}_q||$, and Properties 1 and 2, we can derive:

$$\begin{aligned} ||\varphi_q|| &\leq \overline{c}_q ||\dot{q}||^2 + \overline{d_q} + \overline{j}(||\ddot{q}_d|| + ||\Phi_q|| ||\dot{e}_q||) \\ &\quad + \overline{c}_q ||\dot{q}|| (||\dot{e}_q|| + ||\Phi_q|| ||q||) \\ &\leq K_{q0}^* + K_{q1}^* ||\xi_q|| + K_{q2}^* ||\xi_q||^2, \end{aligned} \quad (13)$$

where $K_{q0}^* \triangleq \overline{c}_q ||\dot{q}_d||^2 + \overline{d}_q + \overline{j} ||\ddot{q}_d||$, $K_{q1}^* \triangleq \overline{c}_q ||\dot{q}_d|| (3 + ||\Phi_q||) + \overline{j} ||\Phi_q||$, and $K_{q2}^* \triangleq \overline{c}_q ||\dot{q}_d|| (2 + ||\Phi_q||)$ are finite *unknown* scalars.

The inner loop controller is proposed as

$$\tau_q = \frac{1}{\gamma_m}\left(-\Lambda_q s_q - \rho_q \frac{s_q}{||s_q||}\right), \quad (14a)$$

$$\rho_q = \frac{1}{1 - E}(\hat{K}_{q0} + \hat{K}_{q1}||\xi_q|| + \hat{K}_{q2}||\xi_q||^2), \quad (14b)$$

where Λ_q is a positive definite user-defined gain matrix, and $\hat{K}_{qi}$ (for $i = 0, 1, 2$) are the estimates of K_{qi}^* governed by the adaptive law

$$\dot{\hat{K}}_{qi} = ||s_q|| ||\xi_q||^i - \alpha_{qi} \hat{K}_{qi}, \ \hat{K}_{qi}(0) > 0, \quad (15)$$

with $\alpha_{qi} \in \mathbb{R}^+$, $(i = 0, 1, 2)$ being user-defined scalars.

The closed-loop stability result is stated below

Theorem 1. *Under Properties 1-3 and Assumption 1, the closed-loop trajectories in (6) and (12), using control* *laws (8), (14), and adaptive laws (9), (15), are Uniformly Ultimately Bounded.*

Proof: See [22].

Remark 5. *In practice, the terms $s_p/||s_p||, s_q/||s_q||$ in control laws (8a) and (14a) continuous are replaced with continuous functions [17], [23], [24] without altering the stability result.*

C. Control Allocation and Overall Control Framework

The control allocation process maps the control inputs vector $u = [u_1, \tau_q] = [u_1, u_2, u_3, u_4]$ (u_1 is determined from τ_p and U as discussed below (9)) to the vector of motor speeds $\Omega = [\omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2]$ via the standard process of minimizing a quadratic function [14]:

$$\varkappa = \arg\min_{\Omega} \Omega^\top L \Omega \quad \text{s.t.} \quad u = B\Omega, \tag{16}$$

$$\text{where } B = \begin{bmatrix} k_f & k_f & k_f & k_f \\ lk_f & -lk_f & -lk_f & lk_f \\ lk_f & -lk_f & lk_f & -lk_f \\ k_\tau & k_\tau & -k_\tau & -k_\tau \end{bmatrix} \tag{17}$$

is the control effectiveness matrix governed by the configuration of the quadrotor. Herein, l is the distance between the rotors and the center of gravity; $k_f$is the thrust force factor determined by the propeller's geometric characteristics, and k_τ is a drag torque factor [12].

The solution of (16) is obtained using a weighted pseudo-inverse:

$$\Omega = \left[LB^T(BLB^T)^{-1}\right] u, \tag{18}$$

where L is defined in (4).

The overall control framework is shown in Fig. 2.

IV. Simulation Results and Analysis

The efficacy of the proposed controller was evaluated using the Gazebo simulation environment, leveraging the DFAutopilot Simulation framework[1] within ROS, and employing the Iris quadrotor model (with the quadrotor's mass, including a payload, established at 1.6 kg). The performance of the proposed adaptive controller is juxtaposed with that of the Dual Adaptive Fault-Tolerant Control [16] (henceforth referred to as DAFTC control) integrated into the DFAutopilot framework.

A. Simulation Scenario and Parameter Selection

The quadrotor was assigned the task of following a lemniscate trajectory. To thoroughly assess the performance of the proposed design, five different simulation scenarios were established, incorporating all fault cases as depicted in Tables II and III. For all scenarios, the control parameters of the proposed controller are selected to be: $\Phi_p = \text{diag}\{1.0, 1.0, 1.1\}$, $\Phi_q = \text{diag}\{1.5, 1.5, 1.5\}$, $\Lambda_p = \text{diag}\{0.95, 0.95, 5.0\}$, $\Lambda_q = \text{diag}\{2.5, 2.0, 2.5\}$, $\hat{K}_{p0}(0) = \hat{K}_{p1}(0) = 0.01$, $\hat{K}_{q0}(0) = \hat{K}_{q1}(0) = \hat{K}_{q2}(0) = 0.1$, $\alpha_{p0} = \alpha_{p1} = 10$, $\alpha_{q0} = \alpha_{q1} = \alpha_{q2} = 1$,$\gamma_m = 0.5$ and, $\varpi_p = 0.1$, $\varpi_q = 1$. For a fair comparison, sliding variables required in DAFTC are selected same as in (5) and (11). The other control parameters for DAFTC are selected as $K_{c1} = \text{diag}\{20.0, 20.0, 10.0\}$, $K_{c2} = \text{diag}\{100.0, 100.0, 25.0\}$, $K_{c3} = \text{diag}\{10.0, 10.0, 8.0\}$, $\eta = \text{diag}\{0.1, 0.1, 0.1\}$, $\beta = \text{diag}\{7.5, 7.5, 5.0\}$ and $\lambda = \text{diag}\{10.0, 10.0, 5.0\}$ (cf. [16] for their definitions), which are optimized for the Iris model. Initial position and attitude for the quadrotor are selected to be $x(0) = 3, y(0) = 0$, $z(0) = 3$ and $\phi(0) = \theta(0) = \psi(0) = 0$.

The performances of the controllers are illustrated in Fig. 3-4, and in Tables II and III via Root Mean Square (RMS) error for various fault cases. We tested the controllers upto 60% loss of efficiency in a single motor. It can be noted from Fig. 3 that the proposed controller can compensate for such fault, whereas the DAFTC controller fails to adapt and crashes (cf. Fig. 4). The RMS errors in Tables II and III, indicate that the proposed controller consistently delivers remarkable control performance. Further, the proposed controller is able to significantly outperform the DAFTC in the case of loss-of-efficiency in multiple motors while negotiating the sharp turns in the desired trajectory. This error is seen to increase with the increase in the number of rotors which suffer a loss of efficiency and the amount of the loss for the DAFTC.

V. Conclusions

An adaptive FTC strategy for quadrotors was proposed, which can tackle actuator loss of efficiency under unknown (state-dependent) system dynamics and external disturbances. Loss-of-efficiency parameter was introduced along with control allocation reconfiguration. The effectiveness of the proposed controller was established against the state-of-the-art method under various scenarios using Gazebo simulation using the DFAutopilot Simulator framework.

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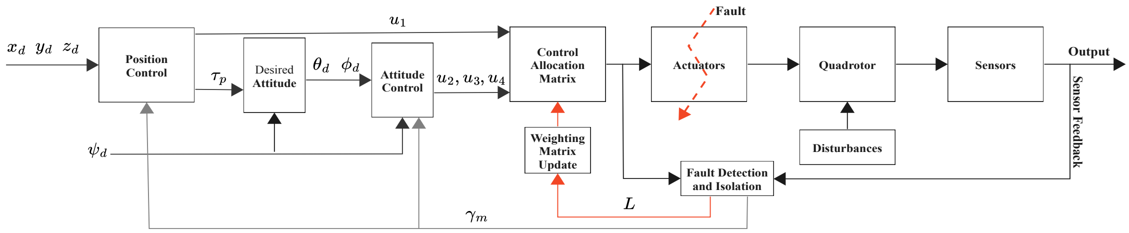

Fig. 2. Schematic of the proposed control framework

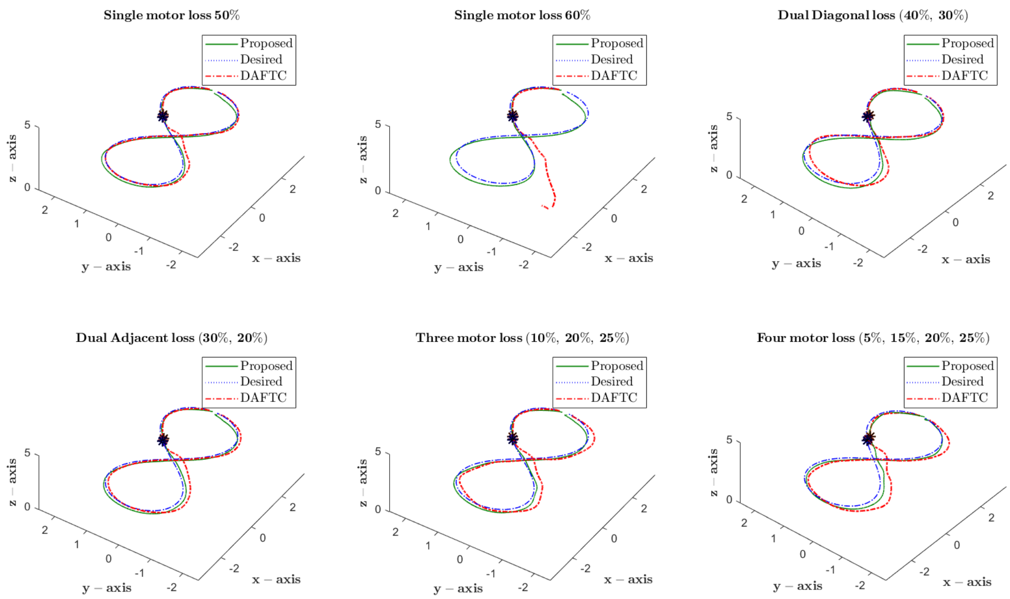

Fig. 3. Path tracking performance comparison (* denotes fault occurrence)

TABLE II

TRACKING PERFORMANCE OF THE PROPOSED CONTROLLER (m_i DENOTES i^{th} MOTOR)

	Motor Efficiency Loss (%)				RMS error before fault			RMS error after fault			RMS error before fault			RMS error after fault		
	m_1	m_2	m_3	m_4	$x(m)$	$y(m)$	$z(m)$	$x(m)$	$y(m)$	$z(m)$	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$
single motor loss	50	-	-	-	0.16	0.13	0.04	0.19	0.15	0.09	0.34	0.21	0.03	0.26	0.21	0.02
single motor loss	60	-	-	-	0.15	0.10	0.04	0.21	0.15	0.08	0.38	0.21	0.02	0.25	0.21	0.21
dual diagonal motors	40	30	-	-	0.13	0.13	0.04	0.20	0.14	0.06	0.36	0.16	0.05	0.24	0.22	0.03
dual adjacent motors	30	-	20	-	0.15	0.13	0.04	0.19	0.13	0.04	0.36	0.19	0.05	0.25	0.22	0.02
three motor loss	-	10	20	25	0.15	0.12	0.04	0.23	0.17	0.05	0.34	0.22	0.04	0.26	0.23	0.02
four motor loss	5	15	20	25	0.16	0.14	0.04	0.19	0.17	0.07	0.40	0.21	0.03	0.27	0.22	0.02

TABLE III

TRACKING PERFORMANCE OF DAFTC CONTROLLER (m_i DENOTES i^{th} MOTOR)

	Motor Efficiency Loss (%)				RMS error before fault			RMS error after fault			RMS error before fault			RMS error after fault		
	m_1	m_2	m_3	m_4	$x(m)$	$y(m)$	$z(m)$	$x(m)$	$y(m)$	$z(m)$	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$	$\phi(deg)$	$\theta(deg)$	$\psi(deg)$
single motor loss	50	-	-	-	0.18	0.21	0.08	0.31	0.16	0.11	0.39	0.28	0.08	0.29	0.26	0.21
single motor loss	60	-	-	-	0.19	0.24	0.08	0.53	0.17	1.38	0.45	0.24	0.1	10.7	18.0	1.95
dual diagonal motors	40	30	-	-	0.22	0.23	0.08	0.34	0.41	0.13	0.59	0.68	0.14	0.35	0.36	0.16
dual adjacent motors	30	-	20	-	0.22	0.24	0.08	0.34	0.22	0.15	0.45	0.29	0.11	0.26	0.26	0.19
three motor loss	-	10	20	25	0.20	0.21	0.07	0.30	0.24	0.16	0.42	0.32	0.08	0.26	0.28	0.22
four motor loss	5	15	20	25	0.21	0.25	0.08	0.31	0.35	0.19	0.44	0.28	0.37	0.26	0.25	0.21

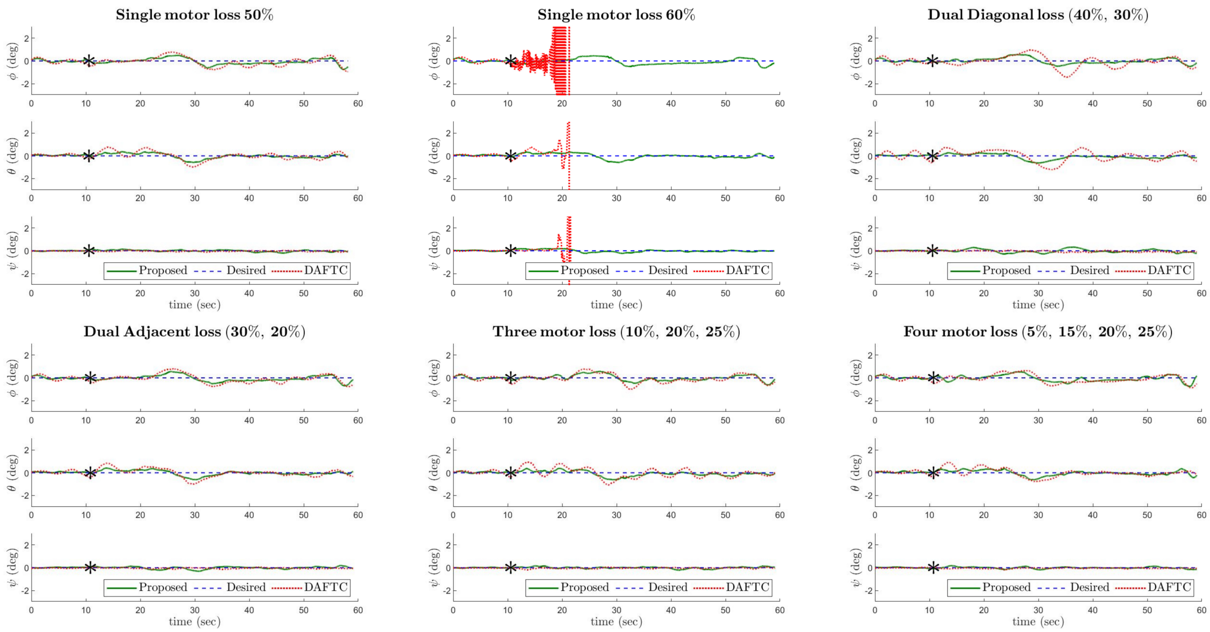

Fig. 4. Attitude tracking performance comparison (* denotes fault occurrence)

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